

Math 320
Fall 04

Projects

Projects may be done alone or with a partner. The goals of the project are to explore a topic that is related to the course and is of interest to you, to learn new mathematics (mostly) on your own, and to design an interesting presentation of your work for your classmates (and possibly others). Listed below are several project ideas. Note that there are many different types of projects. Some involve hands-on work, some involve computer work, and some involve study. Some projects require reading ahead, and some projects require more mathematics background than others. Please note that I already have selected readings for each project; you are of course welcome to find more, but doing research is not the primary purpose of the project. You can also suggest projects that are not on the list.

I will negotiate a scoring rubric with each group so that expectations for the project are agreed upon and clear.

1. Calendar Formula

What day of the week was July 4, 1776? There are several different formulas that can be derived using number theory to determine the day of the week, given any date. This project will involve learning how to derive the formulas, writing an Excel spreadsheet to implement them, and explaining your work to your classmates.

2. Photographing Wallpaper and Frieze Groups

In this project you will attempt to photograph patterns in the real world that correspond to all seventeen wallpaper and all seven frieze groups. You can look at manhole covers, parquet floors, chain link fences, etc. In your poster, you'll display the photographs and include some diagrams to show how you've classified them.

3. Understanding Wallpaper and Frieze Groups in More Depth

Why is it that any wallpaper pattern that has sixfold centers of rotation must also have twofold and threefold centers of rotation? There's a lot more detail underlying these groups than we'll have time for in class. This project will involve reading some material, using Kaleidomania software, and then helping your classmates appreciate the groups in more depth.

4. Sliding Block Puzzles

Have you seen the sliding block puzzle where there are fifteen numbered blocks and one blank space? The one with an oval track where you have to arrange the numbers in order? This project involves special software to analyze the puzzles using group theory and some reading. You'll learn a lot about permutations. A lot of thinking and a lot of fun.

5. Public Key Cryptography

Public Key Cryptography is a relatively recent (but very widely used) method of coding. The key for coding is published so everyone can see it, yet knowing this key

does not allow people to decode coded messages. (This is very different than an A=1, B=2, kind of code). How does it work? Number theory! This project will involve mostly study and then teaching your classmates the basics (computer work can also be included).

6. Error Correcting Codes

This project extends the check digit schemes we'll look at in class. Here, not only can you find out if someone made a mistake in sending a message, but you can also fix the mistake! Uses matrices and base two. This project also involves mostly study and teaching your classmates (computer work can also be included).

7. Mathematical Origami

I have several different ideas for projects involving mathematical origami. All will involve making models and analyzing their symmetries and other properties.

8. Analyzing Regular and Semi-Regular Polyhedra

Just as the wallpaper groups fall into a small number of types, the possible symmetries of a polyhedron fall into an even smaller number of types. The proof of why is quite interesting and elegant. This project involves some study and some building of models using manipulatives (no accurate folding or gluing required) and teaching your classmates.

9. Folding Polygons and Stars out of Adding Machine Tape

There is a lot of number theory involved in this project. You can make a perfectly symmetric 11-pointed star by folding one strip of adding machine tape. Which shapes are possible and which aren't? Find out. This project involves reading, hands-on work, and teaching your classmates.

10. Cycles of Time

Why is a week seven days long? The length of a month and a year were originally determined by natural cycles, but the week is a human creation, and there are actually some cultures that have "weeks" that are of different lengths. This project involves studying the Mayan and Balinese calendars; it uses lots of modular arithmetic.

11. Primality Testing and PseudoPrimes

How does a computer check whether a big number is prime? Not by trying small factors or using a sieve. Some common methods use Fermat's Theorem. Learn about pseudoprimes, Carmichael Numbers, and other new number theory tidbits. Project involves mostly study, with some Excel work, and of course, sharing with classmates.

12. Fermat's Last Theorem

Same Fermat as the one we've studied in class, different theorem (for $n > 2$, $x^n + y^n = z^n$ has no solutions in natural numbers, i.e. the Pythagorean Theorem doesn't work for higher powers). This was one of the most famous unsolved problems in mathematics for hundreds of years; it was finally proved in 1994. The proof is beyond the scope of the

course, but a lot of the ideas are accessible and relevant. Learn what the fuss is about. Mostly study and sharing with classmates.

13. Peg Solitaire

This is the game where you try to jump pegs to leave one left in the middle or to leave some other combination (I can tell you more if you haven't seen it). Which solutions are possible? You can find out using abstract algebra. This project involves some reading ahead.

14. Solving Cubic Equations

One of the historical motivations for the development of abstract algebra was a desire to find analogies to the quadratic formula for higher degree equations. The problem of finding such a formula for degree three equations was solved in the 16th century by some very bizarre people. Also, at that time, they didn't accept negative numbers, which led to some interesting ways of looking at the problem. If you really like manipulating equations, this is a good project for you. Mostly study and sharing with classmates.

15. Bell Ringing

Change ringing is a British pastime where all possible permutations of several bells are rung following specific rules. This project involves reading about change ringing, which involves permutations, groups, and graphs, and doing some kind of presentation, perhaps involving bells borrowed from the music department.

16. Dots and Boxes Game

There is actually quite a bit of strategy to the children's game, "dots and boxes." This project involves reading and leading the class. Here the reading is only marginally connected to the work we're doing in class.

17. Rubik's Cube

This project will be a lot of work, but if you want to connect group theory to learning to solve the Rubik's Cube, this is the project for you. Lots of studying and playing with the cube.

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