

Math 320
Fall 04

Final Exam

You may use a calculator and two sheets of notes. Charts for classifying wallpaper patterns are available. There are 150 points total, please show your work, and justify your reasoning as much as possible.

1. Give two different ways to weigh a 3 oz. sack on a balance scale, using only 4 and 7 oz. weights (10pts).

2. Find all solutions to each of the following congruences (5 pts each):

a. $x + 17 \equiv 42 \pmod{30}$

b. $3y \equiv 6 \pmod{15}$

c. $6z \equiv 9 \pmod{7}$

d. $4a \equiv 3 \pmod{6}$

3. Show that if n is an even integer, then $n(n+1)(n+2)$ is divisible by 24 (10 pts).

4. Represent each of the following in the indicated base (show work) 5pts each:

a. 19_{ten} in base 2:

b. 213_5 in base ten:

c. $\frac{1}{3}$ (base ten) in base 3:

d. $\frac{3}{8}$ (base ten) in base 4:

5. Explain why each of the following is or is not a group (5 pts each):

a. $\{6,12,18,24\}$ under multiplication, mod 30.

b. $\{10,15,20,25\}$ under multiplication mod 30.

c. $3\mathbb{Z}$, the integer multiples of 3, $\{\dots-6,-3,0,3,6,\dots\}$, under addition.

d. $3\mathbb{Z}$, under multiplication.

6. Let R be the symmetry group of a rectangle.

a. Describe the elements of R (5 pts).

b. Describe the operation of R (5 pts).

c. Make a Cayley table for R (i.e. a multiplication table for the group, under the operation). (15 pts)

d. Is R an Abelian group? (i.e. is the operation commutative?) Explain (5 pts).

7. Prove that in a group, if $(ab)^2 = a^2b^2$ for all a, b in the group, then the group is Abelian (i.e. $ab = ba$ for all a, b in the group). (10 pts)

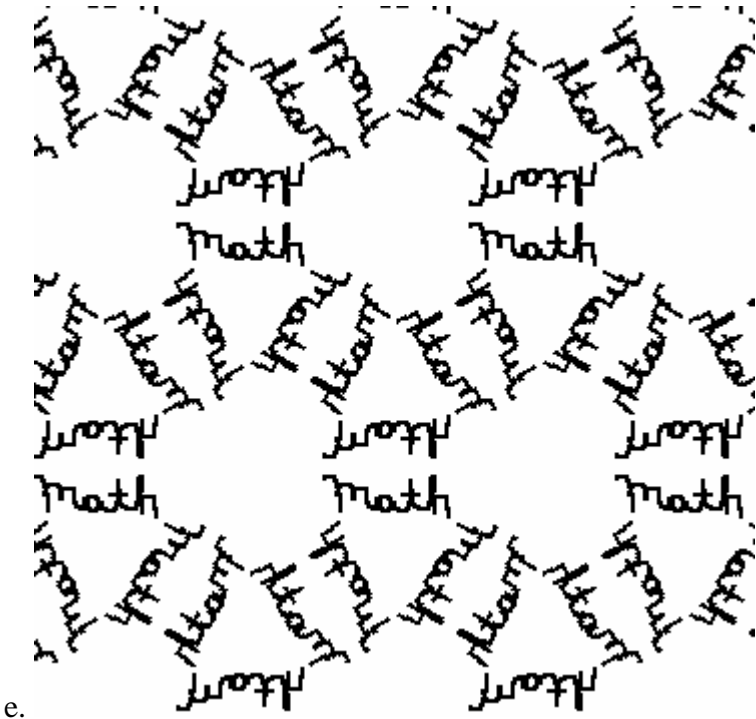
8. Classify the following wallpaper patterns (5 pts each):

a.

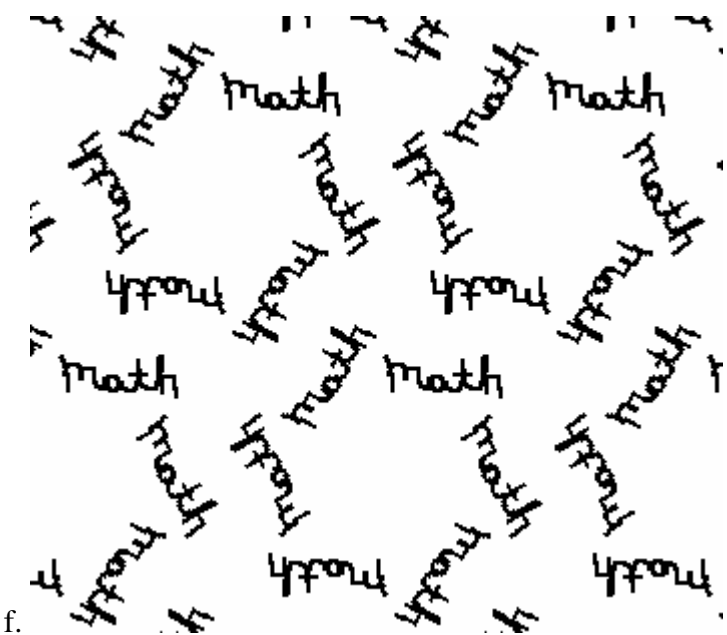
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b.

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e.



f.

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