Math 130
Fall 03

Looking for Fibonacci Patterns

The Fibonacci number sequence; 1, 1, 2, 3, 5, 8, 13, 21, 34, …; starts with two 1's, and then each subsequent term is the sum of the two previous terms. This sequence is famous for many reasons (there's even a journal called *The Fibonacci Quarterly*); one reason is that the sequence contains many, many patterns.

Here are some suggestions for finding patterns in the Fibonacci's. Feel free to ignore them all and just look on your own – maybe you'll find something totally new! You can use Excel, a calculator, or just pencil and paper to explore patterns.

1. Compare the ratios between consecutive terms, i.e. 1/1, 2/1, 3/2, etc. what do you notice? Now try their reciprocals (1/1, ½, 2/3, 3/5, etc.).

2. Look for patterns in the partial sums of the Fibonaccis (i.e. 1, 1+1, 1+1+2, 1+1+2+3, 1+1+2+3+5, etc.). Setting up this pattern on Excel is a good exercise.

3. Try multiplying terms that are one apart in the sequence , i.e. 1 x 2, 1 x 3, 2 x 5, 3 x 8, etc. Look for patterns. Try squaring numbers in the sequence.

4. Look for patterns with odds and evens, with multiples of three, etc.

5. Try adding up ten consecutive Fibonacci numbers. Compare to multiplying various Fibonacci numbers by 11 (there's a nice "magic" trick in here, ask for more details).

6. Try adding up partial sums of every other term. Look for patterns. What if you start at the second term in the sequence instead of the first?

7. The square on the left has area 64, but when the square is cut along the lines and rearranged to make the rectangle on the right, the area becomes 65. What's going on? Hint – see #3 above. Can you make a similar puzzle w/different side lengths?