Math 151
Fall 03

Logarithms

The "log" button on your calculator finds the base 10 logarithm of a number. If $\log(y) = x$, then $10^x = y$.

1. Find the (base 10) logarithms of 10, 100, 1000, 10,000 etc. Find the logarithms of 0.1, 0.01, 0.001, 0.0001, etc.

What patterns do you notice? Reconcile your patterns with the definition of logarithms.

2. Find the base 10 logarithms of 2, 20, 200, 2000, 20,000 0.2, 0.02, 0.002 etc.

What patterns do you notice? Try some other numbers and see whether your patterns hold for them too. Make a conjecture (or several) based on your pattern(s).

3. Find $10^{\log(2)}$ and repeat for other examples you've tried. Make a conjecture about the value of $10^{\log(y)}$. Use the definition of logarithms to confirm or deny your conjecture.

4. Show that $\log\left(10^a\right) = a$. Reconcile with earlier conjectures.

5. Show that $\log(x) + \log(y) = \log(xy)$. Hint: let $x = 10^a$ and $y = 10^b$. Relate to patterns you found earlier.

Note that the property in this exercise is the key property of logarithms. Before calculators and computers, people used logarithms to reduce multiplying large numbers to addition problems. Slide rules work because of logarithms (if you don't know what a slide rule is, look it up – not as ancient as you might think! This author learned to use one in high school in the '70's).

6. Show that $\log\left(a^2\right) = 2\log(a)$; hint: let $a^2 = a \cdot a$, and use the previous exercise. Find similar expressions for $\log\left(a^3\right)$, $\log\left(a^4\right)$, etc. Make up a conjecture about the value of $\log\left(a^t\right)$.

7. Now we will use all of our results so far to solve the equation we found in computing the fractal dimension of a Sierpinski Gasket, namely $2^d = 3$. By taking logarithms of each side and using your results from #6, show that $d = \dfrac{\log(3)}{\log(2)}$.