

Math 130
Fall 03

Free Tuition 2

The president of Wheelock College has declared a holiday. She is going to introduce a new free tuition game. The Riverway has been closed off for the festivities, and a single row of chairs winds along the pavement. The chairs are numbered 1, 2, 3, etc. Here is how the game works:

First, students line up, each in front of a chair.

On her first “pass,” the president walks down the line of chairs and makes sure that all students are standing.

On her second pass, the president starts at the beginning of the line again, and as she walks by, every other student sits, i.e. students in chairs 2, 4, 6, etc. sit.

On her third pass, the president walks down the line and students in every third position *change*, that is, if a student is standing s/he sits, and if a student is sitting s/he stands. Thus the student in seat 3 was standing, and now sits; the student in seat 6 was sitting, and now stands, etc.

On the fourth pass, every fourth student changes, on the fifth pass every fifth student changes, etc., until the president has made one pass for every student present.

When the president is done, everyone who is still standing gets free tuition.

Problems:

Which students will get free tuition? Be sure you can answer the question when there are at least 500 competitors.

What patterns can you find? Can you convince a skeptic that your patterns will continue?

Use formulas to make a list of winners on a spreadsheet. Can you find more than one way to do this?

Make up a variation of this problem or the original free tuition problem. For example, who is the next-to-last person left standing in the original free tuition problem? If you start in a circle and have two people stand and then one person sit and repeat, who wins free tuition?

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