Math 130
Fall 03

Reasoning: The Staircase Problem

One of the most difficult things you will be doing in Math 130 is learning to justify your reasoning. Justifying reasoning means finding a convincing argument that explains why the patterns you find must hold for all cases of a problem. This handout will carry you through one method, with justification, for finding the number of tiles in a staircase of any height. There are many other ways to solve this problem and to justify your results, and you are encouraged to look for alternatives. Maybe you'll find something completely new!

The two attached pages illustrate a visual method for rearranging staircases into rectangles; several students discovered this method in class. Here is an outline of one way to construct and justify a method for finding the number of tiles in any staircase:

1. Fill in the heights of the staircases and the heights and widths of the rectangles. Look for patterns.
2. Find a way to start with a staircase and then determine the height and the width of the rectangle formed after rearranging it. Your method should work for any staircase. You may wish to separate odd and even cases. What would be the height and width of the rectangle formed from a staircase of height 10? 11? 100? 101? 444?
3. Express your method using an equation or a short series of steps, e.g. starting with the height of the staircase (which you might wish to name with a variable), do the following _________ to get the height and the width of the associated rectangle.
4. Using the pictures, explain why it is possible to rearrange every staircase into a rectangle of the shape you describe. This is the key step in the justification – it links your numerical or algebraic pattern(s) to the actual geometric problem and shows that your method will cover all cases.
5. Use the height and the width of the rectangle to find the number of tiles in the staircase. Express this number clearly (as in step 3).

If you finish the above, here are two pictures to get you thinking about alternate methods and proofs: