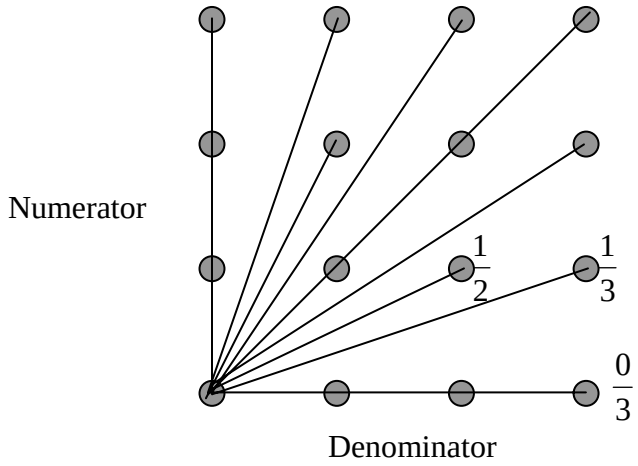
Math 131
Spring 04

Paper 4: Geoboard Fractions

A geoboard is a piece of wood with nails or pegs in it, spaced in a regular way. Many math concepts can be illustrated by stretching rubber bands over the geoboard. You can find several different types of virtual geoboards in the National Library of Virtual Manipulatives at http://matti.usu.edu/nlvm/nav/vlibrary.html. Choose "Geometry" and "Grades 9-12." The geoboards most relevant for this assignment will be the ones marked simply "Geoboard" and the one marked "Coordinate Geoboard." You can also use dot paper, but be sure to use a ruler to make accurate lines.

Most of the Geoboards in our classroom contain 25 nails. We will call these "4-Geoboards," because if the vertical or horizontal distance between two nails in the same row or column represents one unit, then the sides of the entire 4-Geoboard are 4 units long.

Below is a 3-Geoboard, with all possible straight lines drawn connecting the lower left corner to pegs on the board (in the applet, the lines look more like rubber bands). The coordinates of the pegs represent fractions, with numerators the vertical distance from the bottom left corner and denominators the horizontal distance; a few pegs are marked below.



Your first task is to investigate this model for fractions with several different sized geoboards: on each board, make all possible lines that fit within the board. Here are some questions to get you thinking:

- Which fractions are on the same line?
- How does the size of the fractions relate to the position of the lines?
- What fractions are represented on the vertical line?

- Suppose you were trying to figure out which of two fractions was bigger (for example $\frac{5}{7}$ or $\frac{7}{10}$); can you adapt this model?
- Does this model remind you of anything you learned in high school?

For each of the geoboards, make a sequence of the simplified fractions, in order, that can be represented on that board. The 3-sequence is as follows:
$\frac{0}{1}, \frac{1}{3}, \frac{1}{2}, \frac{2}{3}, \frac{1}{1}, \frac{3}{2}, \frac{2}{1}, \frac{3}{1}$.

Some questions to think about:

- What patterns do you notice in each sequence?
- Do you notice any pattern between fractions that are next to each other in a sequence?
- Can you predict, just by looking at the numbers, where new fractions will be added to make the next sequence? For this question and the one above, there are some surprising patterns: see if you can find them!

For your write-up:

1. Introduce the problem. You can introduce it visually, via the geoboards, or numerically, via the sequences. Write for a reader who is at your level in terms of fractions, but has never considered this model or these sequences.
2. Describe what you've found. Use good reasoning to justify your results; if you find a pattern, but are not sure why the pattern must continue, be sure to share both your speculations and your reasons why you think you haven't completely explained all your patterns. Think about what kinds of representations (pictures, tables, equations, etc.) will help you best communicate your ideas.
3. In your conclusion, be sure to relate the problems to other work you've done in mathematics: both in this class and in previous courses.